

3.2 GaAs GaAs has an effective density of states at the conduction CB N_c of $4.7 \times 10^{17} \text{ cm}^{-3}$ and an effective density of states at the VB edge N_v of $7 \times 10^{18} \text{ cm}^{-3}$. Given its bandgap E_g of 1.42 eV calculate the intrinsic concentration and the intrinsic resistivity at room temperature (take as 300 K). Where is the Fermi level? Assuming that N_c and N_v scale as $T^{3/2}$, what would be the intrinsic concentration at 100 °C? If this GaAs crystal is doped with 10^{18} donors cm^{-3} (such as Te), where is the new Fermi level and what is the resistivity of the sample? The drift mobilities in GaAs are shown in Table 3Q2

Table 3Q2

Dopant impurities scatter carriers and reduce the drift mobility (μ_e for electrons and μ_h for holes).

Dopant concentration (cm^{-3})	0	10^{15}	10^{16}	10^{17}	10^{18}
μ_e ($\text{cm}^2 \text{ V}^{-1} \text{ s}^{-1}$)	8500	8000	7000	4000	2400
μ_h ($\text{cm}^2 \text{ V}^{-1} \text{ s}^{-1}$)	400	380	310	220	160

Solution

The intrinsic concentration is

$$n_i = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2k_B T}\right)$$

so that

$$\begin{aligned} n_i &= \left[(4.7 \times 10^{17} \text{ cm}^{-3}) (7 \times 10^{18} \text{ cm}^{-3}) \right]^{1/2} \exp\left[-\frac{1.42 \text{ eV}}{2(8.6174 \times 10^{-5} \text{ eV K}^{-1})(300 \text{ K})} \right] = \\ &= 2.223 \times 10^{12} \text{ m}^{-3} = 2.223 \times 10^6 \text{ cm}^{-3} \end{aligned}$$

The conductivity is

$$\begin{aligned} \sigma &= en_i(\mu_e + \mu_h) \\ &= (1.608 \times 10^{-19} \text{ C}) (2.223 \times 10^6 \text{ cm}^{-3}) (8500 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} + 400 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 3.17 \times 10^{-9} \text{ } \Omega^{-1} \text{ cm}^{-1} \end{aligned}$$

The resistivity is

$$\rho = \frac{1}{\sigma} = \frac{1}{(3.17 \times 10^{-9} \text{ } \Omega^{-1} \text{ cm}^{-1})} = 3.16 \times 10^8 \text{ } \Omega \text{ cm}$$

The energy distance between the bottom of the conduction band E_c and Fermi level of the intrinsic GaAs E_{Fi} is

$$E_c - E_{Fi} = k_B T \ln \left(\frac{N_c}{n_i} \right)$$

so that

$$E_c - E_{Fi} = (8.617 \times 10^{-5} \text{ eV K}^{-1})(300 \text{ K}) \ln \left[\frac{(4.7 \times 10^{17} \text{ cm}^{-3})}{(2.223 \times 10^6 \text{ cm}^{-3})} \right] = 0.675 \text{ eV} .$$

If N_c and N_v scale with temperature as $T^{3/2}$ then the intrinsic concentration at a given temperature T is given by the expression

$$n_i(T) = [N_c(300 \text{ K})N_v(300 \text{ K})]^{1/2} \left(\frac{T}{300 \text{ K}} \right)^{3/2} \exp \left(-\frac{E_g}{2k_B T} \right),$$

so at 100 °C (373 K) we will have

$$\begin{aligned} n_i(373 \text{ K}) &= \left[(4.7 \times 10^{17} \text{ cm}^{-3})(7 \times 10^{18} \text{ cm}^{-3}) \right]^{1/2} \\ &\quad \times \left(\frac{373 \text{ K}}{300 \text{ K}} \right)^{3/2} \exp \left[-\frac{1.42 \text{ eV}}{2(8.6174 \times 10^{-5} \text{ eV K}^{-1})(373 \text{ K})} \right] \\ &= 6.61 \times 10^8 \text{ cm}^{-3}. \end{aligned}$$

For the case when the same GaAs crystal is doped with 10^{18} cm^{-3} donors, one might be tempted to think that $n = N_d$ and to apply the usual expression for the calculation of the Fermi level position:

$$E_c - E_{Fn} = k_B T \ln \left(\frac{N_c}{n} \right) = k_B T \ln \left(\frac{N_c}{N_d} \right)$$

The result from such calculation will be negative (since $N_d > N_c$, degenerate semiconductor), indicating that the Fermi level will be somewhere in the conduction band. Actually, the upper expression is derived on the basis of the assumption that the Fermi level is several kT below the bottom of the conduction band, which allows us to replace the Fermi-Dirac statistics with the more simple Boltzmann statistics, so **it is not applicable** to degenerate semiconductors.

All we can say about the position of the Fermi level for this case is that $E_{Fn} > E_c - kT$. We can very roughly take that $n = N_d$ and estimate the conductivity and the resistivity of the sample,

$$\begin{aligned} \sigma &\approx e N_d \mu_e = (1.608 \times 10^{-19} \text{ C})(1 \times 10^{18} \text{ cm}^{-3})(2400 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 384.5 \Omega^{-1} \text{ cm}^{-1} = 3.845 \times 10^4 \Omega^{-1} \text{ m}^{-1} \end{aligned}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{(384.5 \Omega^{-1} \text{ cm}^{-1})} = 2.6 \times 10^{-3} \Omega \text{ cm} = 2.6 \times 10^{-5} \Omega \text{ m}$$

NOTE

The exact position of the Fermi level and the exact electron concentration can be determined more rigorously applying the Fermi-Dirac statistics. The semiconductor is described by the following equations:

Electron concentration

$$n = N_c F_{1/2} \left(\frac{E_{Fn} - E_c}{k_B T} \right)$$

Hole concentration (Boltzmann statistic, because $E_{Fn} > E_v$)

$$p = N_v \exp \left(\frac{E_v - E_{Fn}}{k_B T} \right)$$

Concentration of the ionized donors

$$N_d^+ = \frac{N_d}{1 + g_d \exp \left(\frac{E_{Fn} - E_d}{k_B T} \right)}$$

No acceptors

$$N_a^- = N_a = 0$$

Charge neutrality

$$N_d^+ + p - n = 0$$

where

$$F_{1/2}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \frac{x^{1/2}}{1 + \exp(x - \eta)} dx$$

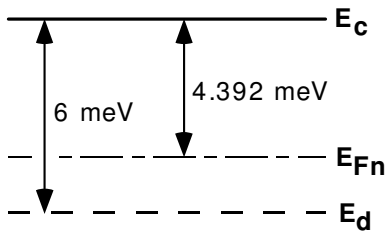
is the Fermi-Dirac integral of order 1/2 and g_d is the donor degeneracy factor.

Assuming donor ionization energy of 6 meV and degeneracy factor of 2 we can determine the position of the Fermi level and the electron concentration by solving the following equation:

$$\frac{N_d}{1 + g_d \exp \left(\frac{E_{Fn} - E_d}{k_B T} \right)} + N_v \exp \left(\frac{E_v - E_{Fn}}{k_B T} \right) - N_c F_{1/2} \left(\frac{E_{Fn} - E_c}{k_B T} \right) = 0$$

The Fermi level position and the corresponding electron concentration are presented in the figure below.

$$n = 3.22 \times 10^{17} \text{ cm}^{-3}$$



We can now calculate the conductivity and resistivity of the sample

$$\sigma = en\mu_e = (1.608 \times 10^{-19} \text{ C})(3.223 \times 10^{17} \text{ cm}^{-3})(2400 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})$$

$$= 124 \text{ } \Omega^{-1} \text{ cm}^{-1} = 1.24 \times 10^4 \text{ } \Omega^{-1} \text{ m}^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{(123.9 \text{ } \Omega^{-1} \text{ cm}^{-1})} = 8.07 \times 10^{-3} \text{ } \Omega \text{ cm} = 8.07 \times 10^{-5} \text{ } \Omega \text{ m}$$

These values are a factor of 2-3 different than the rough estimates calculated above.

3.5 AlGaAs LED emitter An AlGaAs LED emitter for use in a local optical fiber network has the output spectrum shown in Figure 3Q5. It is designed for peak emission at 820 nm at 25°C.

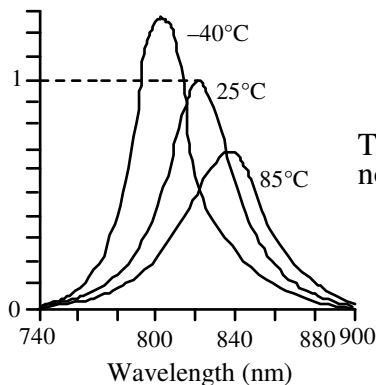
- What is the linewidth $\Delta\lambda$ between half power points at temperatures -40°C , 25°C and 85°C ? What is the empirical relationship between $\Delta\lambda$ and T given three temperatures and how does this compare with $\Delta(h\nu) \approx 2.5k_B T - 3k_B T$?
- Why does the peak emission wavelength increase with temperature?
- Why does the peak intensity decrease with temperature?
- What is the bandgap of AlGaAs in this LED?
- The bandgap, E_g , of the ternary alloys $\text{Al}_x\text{Ga}_{1-x}\text{As}$ follows the empirical expression,

$$E_g(\text{eV}) = 1.424 + 1.266x + 0.266x^2.$$

What is the composition of the AlGaAs in this LED?

- When the forward current is 40 mA, the voltage across the LED is 1.5V and the optical power that is coupled into a multimode fiber through a lens is 25 μW . What is the efficiency?

Relative spectral output power



The output spectrum from GaAlAs LED. Values normalized to peak emission at 25°C.

Figure 3Q5

Solution

a We note that the emitted wavelength is related to the photon energy E_{ph} by

$$\lambda = c/\nu = hc/E_{ph}.$$

If we differentiate λ with respect to photon energy E_{ph} we get

$$\frac{d\lambda}{dE_{ph}} = -\frac{hc}{E_{ph}^2}$$

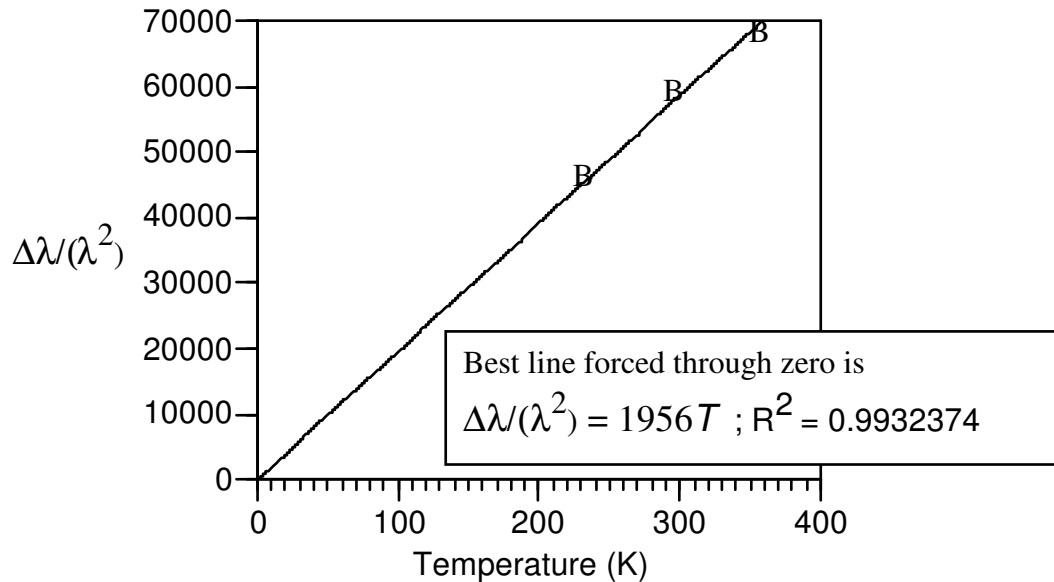
We can represent small changes or intervals (or Δ) by differentials, e.g. $\Delta\lambda/\Delta E_{ph} \approx |d\lambda/dE_{ph}|$, then

$$\Delta\lambda \approx \frac{hc}{E_{ph}^2} \Delta E_{ph}$$

We are given the energy width of the output spectrum, $\Delta E_{ph} = \Delta(h\nu) \approx 3k_B T$. Then, using the latter and substituting for E_{ph} in terms of λ we find,

$$\Delta\lambda \approx \lambda^2 \frac{3k_B T}{hc} \quad \text{or} \quad \Delta\lambda \approx \lambda^2 \frac{3k_B T}{hc}$$

Temperature → Parameter ↓	-40 °C	25 °C	85 °C	Comment
λ_{peak} (nm)	804	820	837	
$\Delta\lambda$ (nm) (Measured)	30	40	48	
$\Delta\lambda$ (nm) (Calculated); $\Delta E_{ph} = 2.5 kT$	26.2	34.8	43.6	
$\Delta\lambda$ (nm) (Calculated); $\Delta E_{ph} = 3 kT$	31.4	41.7	52.3	Very close



The theory predicts that $\Delta\lambda/\lambda^2$ vs T should be a straight line because,

$$\Delta\lambda \approx \lambda^2 \frac{\Delta E_{ph}}{hc}$$

so that
$$\frac{\Delta\lambda}{\lambda^2} = \frac{mk_B T}{hc}$$

where $\Delta E_{ph} = mk_B T$ and m is a numerical constant that represents the ratio $\Delta(h\nu)/(k_B T)$ and is determined from the slope of the $\Delta\lambda/\lambda^2$ vs. T plot.

The three points plotted in the figure seems to follow this behavior. The best line forced through zero has a slope that indicates $m = 2.8$.

b The bandgap decreases with temperature

c There are two factors to consider. (i) Spectral intensity means intensity per unit wavelength, that is, $dI/d\lambda$. The integration of the spectral curve gives the total intensity, the total number of photons emitted per unit area per unit time. As the spectrum broadens with temperature we would naturally expect the peak to decrease with temperature. (ii) Higher the temperature, the stronger are the lattice vibrations (there are more phonons). Indirect or radiationless transitions, those that do not emit photons, require phonons (lattice vibrations) which encourage indirect transitions. Thus increasing the temperature increases indirect transitions at the expense of direct transitions and the light intensity decreases. If (ii) was totally absent then the areas under the curves for all the three spectra would be identical.

d Use the peak emission wavelength to find E_g as follows:

$$\text{At } -40^\circ\text{C (233 K)}, \lambda_{\text{peak}} = 804 \text{ nm.}$$

At 25 °C (298 K), $\lambda_{\text{peak}} = 820 \text{ nm}$.

At 85 °C (358 K), $\lambda_{\text{peak}} = 837 \text{ nm}$.

We first note that we need the required bandgap E_g at the wavelength of interest. The photon energy at peak emission is $hc/\lambda_{\text{peak}} = E_g + k_B T$. Then,

$$E_g = \frac{ch}{e\lambda_{\text{peak}}} - \frac{k_B T}{e}$$

and at $\lambda_{\text{peak}} = 820 \times 10^9 \text{ m}$, taking $T = 25 + 273 \text{ K}$,

$$E_g = \frac{(3 \times 10^8)(6.626 \times 10^{-34})}{(1.6 \times 10^{-19})(820 \times 10^9)} - 0.0257 \text{ eV} = \mathbf{1.4863 \text{ eV}}$$

e The bandgap E_g of the ternary alloys $\text{Al}_x\text{Ga}_{1-x}\text{As}$ follows the empirical expression,

$$E_g(\text{eV}) = 1.424 + 1.266x + 0.266x^2.$$

$\therefore E_g(\text{eV}) = 1.4863 = 1.424 + 1.266x + 0.266x^2.$

Solving for x we find, $x = \mathbf{0.05}$.

f From the definition of efficiency η ,

$$\begin{aligned} \eta &= \frac{\text{Output optical power}}{\text{Input electrical power}} \\ &= \frac{P_o}{IV} = \frac{25 \times 10^{-6} \text{ W}}{(40 \times 10^{-3} \text{ A})(1.5 \text{ V})} = 0.000417 = 0.0417\% \end{aligned}$$

3.6 III-V compound semiconductors in optoelectronics Figure 3Q6 represents the bandgap E_g and the lattice parameter a in the quaternary III-V alloy system. A line joining two points represents the changes in E_g and a with composition in a ternary alloy composed of the compounds at the ends of that line. For example, starting at GaAs point, $E_g = 1.42 \text{ eV}$ and $a = 0.565 \text{ nm}$, and E_g decreases and a increases as GaAs is alloyed with InAs and we move along the line joining GaAs to InAs. Eventually at InAs, $E_g = 0.35 \text{ eV}$ and $a = 0.606 \text{ nm}$. Point X in Figure 3Q6 is composed of InAs and GaAs and it is the ternary alloy $\text{In}_x\text{Ga}_{1-x}\text{As}$. It has $E_g = 0.7 \text{ eV}$ and $a = 0.587 \text{ nm}$ which is the same a as that for InP. $\text{In}_x\text{Ga}_{1-x}\text{As}$ at X is therefore lattice matched to InP and can hence be grown on an InP substrate without creating defects at the interface.

Further, $\text{In}_x\text{Ga}_{1-x}\text{As}$ at X can be alloyed with InP to obtain a quaternary alloy $\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$ whose properties lie on the line joining X and InP and therefore all have the same lattice parameter as InP but different bandgap. Layers of $\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$ with composition between X and InP can be grown epitaxially on an InP substrate by various techniques such as liquid phase epitaxy (LPE) or molecular beam epitaxy (MBE) .

The shaded area between the solid lines represents the possible values of E_g and a for the quaternary III-V alloy system in which the bandgap is direct and hence suitable for direct recombination.

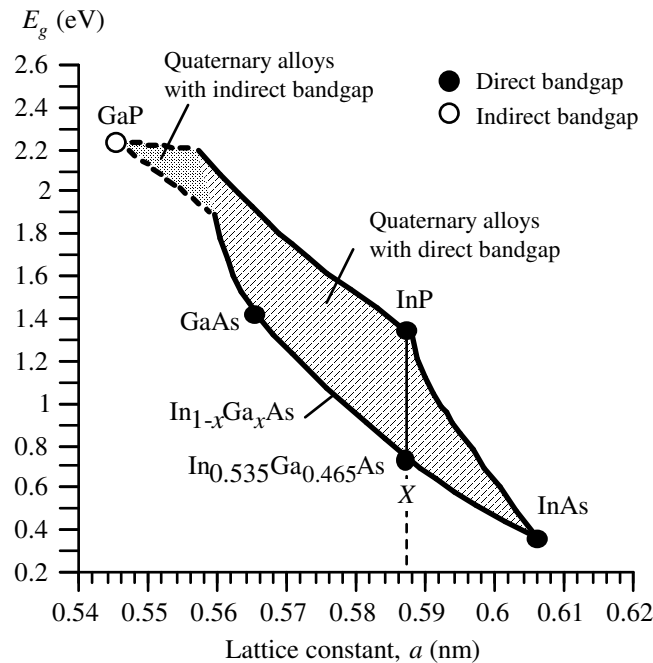
The compositions of the quaternary alloy lattice matched to InP follow the line from X to InP.

a Given that the $\text{In}_x\text{Ga}_{1-x}\text{As}$ at X is $\text{In}_{0.535}\text{Ga}_{0.465}\text{As}$ show that quaternary alloys $\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$ are lattice matched to InP when $y = 2.15x$.

b The bandgap energy E_g , in eV for $\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$ lattice matched to InP is given by the empirical relation,

$$E_g \text{ (eV)} = 1.35 - 0.72y + 0.12 y^2$$

Find the composition of the quaternary alloy suitable for an emitter operating at $1.55 \mu\text{m}$.



Bandgap energy E_g and lattice constant a for various III-V alloys of GaP, GaAs, InP and InAs. A line represents a ternary alloy formed with compounds from the end points of the line. Solid lines are for direct bandgap alloys whereas dashed lines for indirect bandgap alloys. Regions between lines represent quaternary alloys. The line from X to InP represents quaternary alloys $\text{In}_{1-x}\text{Ga}_x\text{As}_{1-y}\text{P}_y$ made from $\text{In}_{0.535}\text{Ga}_{0.465}\text{As}$ and InP which are lattice matched to InP.

Figure 3Q6

Solution

a The III–V quaternary alloy is made of $(\text{In}_{0.535}\text{Ga}_{0.465}\text{As})$ alloyed with InP. Suppose that z fraction of $\text{In}_{0.535}\text{Ga}_{0.465}\text{As}$ is alloyed with $(1 - z)$ fraction of InP.

$z = 0$ for InP and $z = 1$ for $\text{In}_{0.535}\text{Ga}_{0.465}\text{As}$

The formula for the quaternary alloy is $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$.

Thus, $z(\text{In}_{0.535}\text{Ga}_{0.465}\text{As}) + (1-z)\text{InP} = \text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$

Use $x = x'$ to represent point X or $(\text{In}_{0.535}\text{Ga}_{0.465}\text{As})$, that is, $x' = 0.535$ for the In content.

We can now balance the contents for each element.

For P $1 - z = 1 - y \quad \therefore \quad z = y$

For Ga $(1 - x')z = x \quad \therefore \quad y = 2.15x$

For In $x'z + (1 - z) = 1 - x \quad \therefore \quad z = x/(1 - x')$

For As $z = y \quad \therefore \quad y = 2.15x$

b Consider a lattice matched InGaAsP to InP. Given $\lambda = 1.55 \mu\text{m}$.

$$E_g = 1.24/\lambda = 0.80 \text{ eV}$$

Thus, $E_g \text{ (eV)} = 1.35 - 0.72y + 0.12 y^2 = 0.80 \text{ eV}$

$$\therefore \quad 0.12y^2 - 0.72y + 0.55 = 0$$

$$\therefore \quad y = 0.90, x = 0.42$$

$$\therefore \quad \text{In}_{0.58}\text{Ga}_{0.42}\text{As}_{0.9}\text{P}_{0.1}$$

3.8 Linewidth of LEDs Experiments carried out on various direct bandgap semiconductor LEDs give the output spectral linewidth (between half intensity points) listed in Table 3Q8-1. We know that a spread in the wavelength is related to a spread in the photon energy,

$$\Delta\lambda \approx \frac{hc}{E_{ph}^2} \Delta E_{ph} \quad (1)$$

Suppose that we write $E_{ph} = hc/\lambda$ and $\Delta E_{ph} = \Delta(h\nu) \approx mk_B T$ where m is a numerical constant. Show that,

$$\Delta\lambda \approx \lambda^2 \frac{mk_B T}{hc} \quad (2)$$

and by appropriately plotting the data in Table 3Q8-1 find m .

Table 3Q8-1 shows the linewidth $\Delta\lambda_{1/2}$ for various visible LEDs. Radiative recombination is obtained by appropriately doping the material. Using $m \approx 3$, $T = 300$ K, in Eq. (2) calculate the expected spectral width for each and compare with the experimental value. What is your conclusion? Do you think E_N in Figure 3.24 (b) is a discrete level?

Table 3Q8-1

Linewidth $\Delta\lambda_{1/2}$ between half points in the output spectrum (Intensity vs wavelength) of GaAs and AlGaAs LEDs.

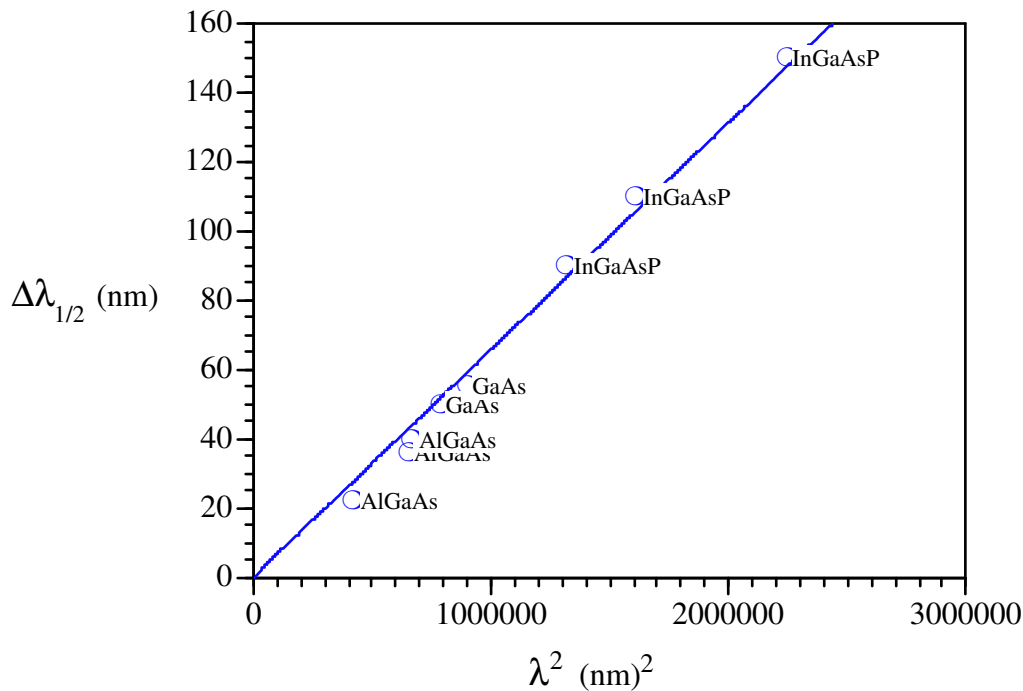
Peak wavelength of emission (λ) nm	650	810	820	890	950	1150	1270	1500
$\Delta\lambda_{1/2}$ nm	22	36	40	50	55	90	110	150
Material (Direct E_g)	AlGaAs	AlGaAs	AlGaAs	GaAs	GaAs	InGaAsP	InGaAsP	InGaAsP

Table 3Q8-2

Linewidth $\Delta\lambda_{1/2}$ between half points in the output spectrum (Intensity vs wavelength) of four various visible LEDs using SiC and GaAsP materials.

Peak wavelength of emission (λ) nm	468	565	583	600	635
$\Delta\lambda_{1/2}$ nm	66	28	36	40	45
Color	Blue	Green	Yellow	Orange	Red
Material	SiC (Al)	GaP (N)	GaAsP (N)	GaAsP (N)	GaAsP

Solution



The best line forced through zero is

$$\Delta\lambda_{1/2} = [6.57 \times 10^{-5} \text{ (nm)}^{-1}] \lambda^2$$

and

$$\text{Slope} = 6.57 \times 10^{-5} \text{ (nm)}^{-1}.$$

This slope is mkT/hc ; thus $m \approx 3.15$. Very close to the theoretically predicted value of $m = 2.5 - 3$.

Table 2Q8-3 shows the calculated spectral widths $\Delta\lambda_{1/2}$ using $m = 3$. The actual observed widths are substantially larger than the expected $\Delta\lambda_{1/2}$ using $m = 3$, as we found above for the direct bandgap materials. If the recombination center were a discrete level, we would expect a spread in the photon energy that is controlled by the energy distribution of holes

(electrons) in the valence (conduction) band, that is about $\sim 1.5k_B T$, or $m = 1.5$. The observed spread is much more than $m = 1.5$ and hence the energy level cannot be discrete. (It is possible to give a semiquantitative plausible explanation as follows. A captured electron will have a wavefunction that is localized and hence a smaller uncertainty in its position than in the band; i.e. Δx will be small. That means the uncertainty Δp in its momentum will be higher and hence the uncertainty in its energy will also be higher. We would expect that the spread of photon energies will be more than from band to band recombinations.)

Table 3Q8-3

Linewidth $\Delta\lambda_{1/2}$ between half points in the output spectrum (Intensity vs wavelength) of four various visible LEDs using SiC and GaAsP materials.

Peak wavelength of emission (λ) nm	468	565	583	600	635
$\Delta\lambda_{1/2}$ nm	66	28	36	40	45
Expected $\Delta\lambda_{1/2}$ nm using $m = 3$	13.7	20.0	21.3	22.5	25.2
Color	Blue	Green	Yellow	Orange	Red
Material	SiC (Al)	GaP (N)	GaAsP (N)	GaAsP (N)	GaAsP

3.10 LED-Fiber coupling Efficiency

a It is found that approximately 200 μW is coupled into a multimode step index fiber from a surface emitting LED when the current is 75 mA and the voltage across the LED is about 1.5 V. What is the overall efficiency of operation?

b Experiments are carried out on coupling light from a 1310 nm ELED (edge emitting LED) in multimode and single mode fibers.

(i) At room temperature, when the ELED current is 120 mA, the voltage is 1.3 V and light power coupled into a 50 μm multimode fiber with NA (numerical aperture) = 0.2 is 48 μW . What is the overall efficiency?

(ii) At room temperature, when the ELED current is 120 mA, the voltage is 1.3 V and light power coupled into a 9 μm single mode fiber is 7 μW . What is the overall efficiency?

Solution

a
$$\eta_{\text{overall}} = \frac{P_o}{IV} = \frac{200 \times 10^{-6} \text{ W}}{(75 \times 10^{-3} \text{ A})(1.5 \text{ V})} = 1.8 \times 10^{-3} = \mathbf{0.18 \%}$$

b (i)
$$\eta_{\text{overall}} = \frac{P_o}{IV} = \frac{48 \times 10^{-6} \text{ W}}{(120 \times 10^{-3} \text{ A})(1.3 \text{ V})} = \mathbf{0.0307 \%}$$

$$(ii) \quad \eta_{\text{overall}} = \frac{P_o}{IV} = \frac{7 \times 10^{-6} \text{ W}}{(120 \times 10^{-3} \text{ A})(1.3 \text{ V})} = \mathbf{0.0045\%}$$